### Mining Approximate Top-K Subspace Anomalies in Multi-Dimensional Time-Series Data

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**VLDB 2007** 

#### Time Series Data

• Many applications produce time series data



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Compare time series to gather differences





# Apple stock has a very different "trend"





2006	Intel stock had	200
Time	different magnitude	

## Find anomalies in a data cube of multi-dimensional time series data

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### Multi-Dimensional Attributes

- Time series are not flat data; contains multi-dimensional attributes
- Stock example
  - Apple and Intel are a part of the NASDAQ Computers Index unter an apple and Intel are a part of the NASDAQ Computers Index
  - Apple is hardware/software; Intel is hardware
  - Related to NASDAQ-100 Technology Stock Index
- Sales example



- Multi-dimensional information collected for every sale (e.g., buyer age, product type, store location, purchase time)
- Compare sales by any combination of categories or sub-categories: "sales of sporting apparel to <u>males with 3+ children</u> have been declining compared to <u>overall male</u> sporting apparel sales"

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- Find anomalies in the data cube of multi-dimensional time series data
- Input data: relation **R** with a set of time series **S** associated with each tuple
  - Attributes of R form a data cube C<sub>R</sub>
  - Each s<sub>i</sub> is a time series
  - Each u<sub>i</sub> is a scalar indicating the count of the tuple

Gender	Education	Income	Product	Profit	Count
Female	Highschool	35k-45k	Food	S <sub>1</sub>	U1
Female	Highschool	45k-60k	Apparel	<b>S</b> <sub>2</sub>	U <sub>2</sub>
Female	College	35k-45k	Apparel	<b>S</b> 3	U <sub>3</sub>
Female	College	35k-45k	Book	<b>S</b> 4	U4
Female	College	45k-60k	Apparel	$S_5$	U5
Female	Graduate	45k-60k	Apparel	<b>S</b> 6	U <sub>6</sub>
Male	Highschool	35k-45k	Apparel	<b>S</b> 7	U7
Male	College	35k-45k	Food	<b>S</b> <sub>8</sub>	U <sub>8</sub>

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Female	College	45k-60k	Apparel	<b>S</b> 5	$U_5$
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Female	Graduate	45k-60k	Apparel	$S_6$	U <sub>6</sub>
Male	Highschool	35k-45k	Apparel	<b>S</b> <sub>7</sub>	U <sub>7</sub>
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Female	College	35k-45k	Book	<b>S</b> 4	U4
Female	College	45k-60k	Apparel	$S_5$	$U_5$
Female	Graduate	45k-60k	Apparel	<b>S</b> 6	U <sub>6</sub>
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- Given a relation R, a data cube (denoted as C<sub>R</sub>) is the set of aggregates from all possible group-by's on R
- In a *n*-dimensional data cube, each cell has the form c = (a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>n</sub> : m) where each a<sub>i</sub> is the value of i<sup>th</sup> attribute and m is the cube measure (e.g., profit)
- A cell is k-dimensional if there are exactly k ( $\leq$  n) values amongst  $a_i$  which are not \* (i.e., all)
  - 2-dimensional cell: (Female, \*, \*, Book: x)
  - 3-dimensional cell: (\*, College, 35k-45k, Apparel: y)
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#### Query Model

- Given R, a probe cell p ∈ C<sub>R</sub>, and an anomaly function g, find the anomaly cells among descendants of p in C<sub>R</sub> as measured by g
  - Each abnormal cell must satisfy a minimum support (count) threshold
  - Anomaly does not have to hold for entire time series
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### Related Work

- Exploratory Data Analysis
  - [Sarawagi SIGMOD'00] explores OLAP anomaly but necessitates full cube materialization
  - [Palpanas SSDBM'01] approximately finds interesting cells in data cube but still requires exponential calculations
  - [Imielinski DMKD'02] requires anti-monotonic measure and does not focus on time series
- Time Series Data Cube [Chen VLDB'02]
  - Only suitable for low-dimensional data
  - Requires user guidance
- General outlier detection, subspace clustering, and time series similarity search does not address OLAP-style data

### Measuring Anomaly: Intuition

1. For every cell, compute the expected time series (with respect to the probe cell)

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- 3. Rank to get top k

#### **Observed Time Series**

- Given any cell *c* in C<sub>R</sub>, there is an associated observed time series *s<sub>c</sub>*
- In the context of a probe cell *p*, it is computed by aggregating all time series associated with both *c* and *p*

$$s_c = \sum_{tid_i \in (c \cap \sigma_p(R))} s_i$$

### Observed Time Series (2)

Gender	Education	Income	Product	Profit	Count
Female	Highschool	35k-45k	Food	S <sub>1</sub>	U1
Female	Highschool	45k-60k	Apparel	<b>S</b> 2	150
Female	College	35k-45k	Apparel	<b>S</b> 3	200
Female	College	35k-45k	Book	<b>S</b> 4	U4
Female	College	45k-60k	Apparel	$S_5$	600
Female	Graduate	45k-60k	Apparel	<b>S</b> 6	50
Male	Highschool	35k-45k	Apparel	<b>S</b> 7	U <sub>7</sub>
Male	College	35k-45k	Food	S <sub>8</sub>	U <sub>8</sub>

• Example: *p* = (Gender = "Female", Product = "Apparel")

		C	Sc	c	
	Education	Education Income		Count	
0	*	*	$S_2 + S_3 + S_5 + S_6$	1000	
	Highschool	*	<b>S</b> 2	150	
	College	*	$S_3 + S_5$	800	

#### **Expected Time Series**

- Given any cell c that is a descendant of p, there is also an expected time series ŝ<sub>c</sub>
- Intuition: A descendant cell of p is a subset of p. Assuming that market segments behave proportionally by its size, one can calculate the expected time series from p's time series

$$\hat{s}_c = \left(\frac{|c|}{|p|}\right) s_p$$

С		Sc	Ŝc	c
Education	Income	Prof	Count	
*	*	$S_2 + S_3 + S_5 + S_6 = S_p$	n/a	1000
Highschool	*	S <sub>2</sub>	150 / 1000 x s <sub>p</sub>	150
College	*	$S_3 + S_5$	800 / 1000 x s <sub>p</sub>	800

#### Anomaly Definition

• General idea:  $g(s_c, \hat{s}_c) \Rightarrow R$ 

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- Four types of anomalies
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  - Magnitude
  - Phase
  - Miscellaneous



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- General idea:  $g(s_c, \hat{s}_c) \Rightarrow R$
- Four types of anomalies
  - Trend
  - Magnitude
  - Phase
  - Miscellaneous
- Measured via first-order linear regression
  - Simple and efficient (direct cube aggregation of parameters [Chen VLDB'02])
  - Effective at catching obvious anomalies



#### Mining Top-K Anomalies in Data Cube

**Algorithm 1** Naïve Top-k Anomalies

Input: Relation R, time-series data S, query probe cell p, anomaly function g, parameter k, minimum support mOutput: Top-k scoring cells in  $C_p$  as ranked by g and satisfies m

- 1. Retrieve data for  $\sigma_p(R)$
- 2. Compute the data cube  $C_p$  with  $\sigma_p(R)$  as the fact table with m as the iceberg parameter
- 3. Return top k anomaly cells in  $C_p$  for each g

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1. Expensive to compute C<sub>p</sub> (exponential in number of dimensions)

2. Finds all anomalies before collecting top-*k* 

• Subspace Iterative Time Series Anomaly Search (SUITS)



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• Iteratively select subspaces out of the 2<sup>n</sup> total subspaces



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- Iteratively select subspaces out of the 2<sup>n</sup> total subspaces
- Compute anomalies within subspaces



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- Iteratively select subspaces out of the 2<sup>n</sup> total subspaces
- Compute anomalies within subspaces
- Combine to form overall anomalies



- Intuition
  - By definition, anomalies are rare and most of the 2<sup>n</sup> subspaces do not contain any
  - Descendant cells stemming from the same anomalies (in some ancestor cell) should exhibit similar abnormal behavior
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  - 1. Search for a group of similar anomalies in the set of base cells
  - 2. Find a subspace correlated with the group
  - 3. Compute the local top-k anomalies in the subspace



#### • Time Anomaly Matrix



 Table 4: Time Anomaly Matrix

- Partition each observed and expected time series into subsequences and compute anomalies
- Group anomalies by type and also amount
- Iteratively select groups of similar anomaly cells from matrix

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		Time	Weature	Weenro	
Highschool	45k-60k	None	Magnitude	Magnitude	
		Mature	Weature	Portugation of the second seco	
College	35k–45k	Phase	None	Misc	
		Provide the second seco	Weight	Time	
College	45k-60k	Phase	Magnitude	Magnitude	
		Weature	Weight	Weight	
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- Given a group in the Time Anomaly Matrix, select its correlated subspace
- Rank attribute-value pairs by Anomaly Likelihood (AL) score
  - Attribute values that occur very frequently and within a homogenous dimension have high AL scores
  - ► AL = (Frequency of Attribute-Value) x (Entropy of Attribute)<sup>-1</sup>
- Select the top few and form the candidate subspace

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		W	energy Time	Time
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		W	engreger de la construction de l	W
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Highschool	$^{4}$ Attrib	ute Value	Frequency AL	Score
	Incom	e = 45k - 60k	3 Time	Time
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### Discovering Top-K Anomaly Cells

- Each subspace is small enough (~5 dimensions) for full cube materialization
- Efficient Regression Calculation
  - Linear regression needed for anomaly calculation (comparisons between parameters of observed and expected time series regression)
  - Regression parameters can be **aggregated losslessly** [Chen VLDB'02]
  - Only need to perform regression calculation once in the base cuboid
  - Higher level cuboids' regression parameters can be calculated via simple aggregation

### Discovering Top-K Anomaly Cells (2)

- More efficient top-k anomaly detection (i.e., avoid computing the whole data cube)
- Intuition: calculate anomaly upper bounds during cubing and prune branches if upper bound is below current top-k
- Procedure
  - Bottom-up cube calculation [Beyer SIGMOD'99]



#### SUITS Algorithm in Summary

#### Algorithm 2 SUITS

Input & Output: Same as Algorithm 1

- 1. Retrieve data for  $\sigma_p(R)$
- 2. Repeat until global answer set contains global top-k
- 3.  $B \leftarrow \text{candidate attribute values from } \{A_1, \ldots, A_n\}$
- 4. Retrieve top k anomaly cells from  $C_B$  using g and m
- 5. Add top k cells to global answer set
- 6. Remove discovered anomalies from input
- 7. Return top k cells in global answer set
- Final top-*k* is approximation of true global top-*k*
- Top-*k* pruning relies on monotonic properties of upper bound. If not satisfied, need to compute full subspace cube

#### Experiments

- Real market sales data from industry partner
- Time series data from 1999 to 2005
- Nearly 1 million sales and 600 dimensions

#### Sample Query 1



- **Probe**: Gender = "Male" ^ Marital = "Single" ^ Product = luxury item
- **Greatest anomaly**: <u>Generation = "Post-Boomer" : less than expected</u>
- Explanation: "Post-Boomer" are young and do not have enough money yet to purchase luxury item

#### Sample Query 2

- **Probe**: Gender = "Female" ^ Education = "Post-Graduate" ^ Product = cheap item
- Greatest anomaly:
  - 1. Employment = "Full-Time"  $\Rightarrow$  less
  - 2. Occupation = "Manager/Professional"  $\Rightarrow$  less
  - 3. <u>Number of Children Under  $16 = 0 \Rightarrow more</u>$ </u>
- **Explanation**: Number of Children Under  $16 = 0 \Leftrightarrow$  "Young"  $\Leftrightarrow$  not enough accumulated wealth



### Query Efficiency

Probe	R	Naïve	S		SUITS		Common Top-10
		Time	Time	% Improve	Time	% Improve	
Male, Single	10	14	5.9	58%	5.4	61%	9
Male, Married	10	299	95	68%	60	80%	10
Male, Divorced	10	3.6	2.8	22%	2.8	22%	10
Female, Single	10	15	8.2	46%	7.0	53%	9
Female, Married	10	114	31.0	73%	23.0	80%	8
Female, Divorced	10	5.5	3.8	31%	3.7	33%	10
Post-Boomer, Children=0	11	68.8	39.6	43%	32.1	53%	10
Post-Boomer, Children=1	11	16.8	5.4	68%	4.8	71%	10
Post-Boomer, Children=2	11	15.5	7.8	50%	6.7	57%	10
Boomer, Children=0	11	108.9	75.7	30%	52.4	52%	10
Boomer, Children=1	11	120.3	68.9	43%	58.0	52%	10
Boomer, Children=2	11	46.6	27.2	42%	23.6	49%	10
Average			48%		55%	9.6	

Table 8: Run times of trend anomaly query with low dimensional data ( $10 \le |R| \le 11$ )

### Dimensionality Efficiency



Figure 9: Running time vs. number of dimensions

### Conclusion

- Detecting anomalies in data cube of time series data
- Iterative subspace search
- Efficient top-*k* anomaly detection
- Experiments with real data

Thank You!