Multi-Probe LSH: Efficient Indexing for High-Dimensional Similarity Search

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Motivations

- Massive amounts of feature-rich data
  - Audio, video, digital photos, sensor data, …
- Fuzzy & high-dimensional
  - Similarity search in high dimensions
  - KNN or ANN in feature-vector space
- Important in various areas
  - Databases, data mining, search engines …

Ideal Indexing for Similarity Search

- Accurate
  - Return results that are close to brute-force search
- Time efficient
  - O(1) or O(log N) query time
- Space efficient
  - Small space usage for index
  - May fit into main memory even for large datasets
- High-dimensional
  - Work well for datasets with high dimensionality

Previous Indexing Methods

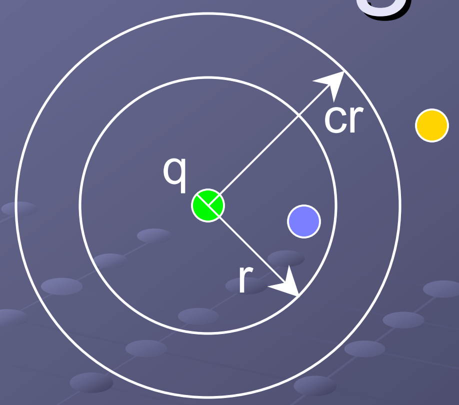
- K-D tree, R-tree, X-tree, SR-tree …
  - "curse of dimensionality"
  - Linear scan outperforms when d > 10 *[WSB98]*
- Navigating nets *[KL04]*, cover tree *[BKL06]*
  - Based on "intrinsic dimensionality"
  - Do not perform well with high intrinsic dimensionality
- Locality sensitive hashing (LSH)

Outline

- Motivations
- Locality sensitive hashing (LSH)
  - Basic LSH, entropy-based LSH
- Multi-probe LSH indexing
  - Step-wise probing, query-directed probing
- Evaluations
- Conclusions & future work

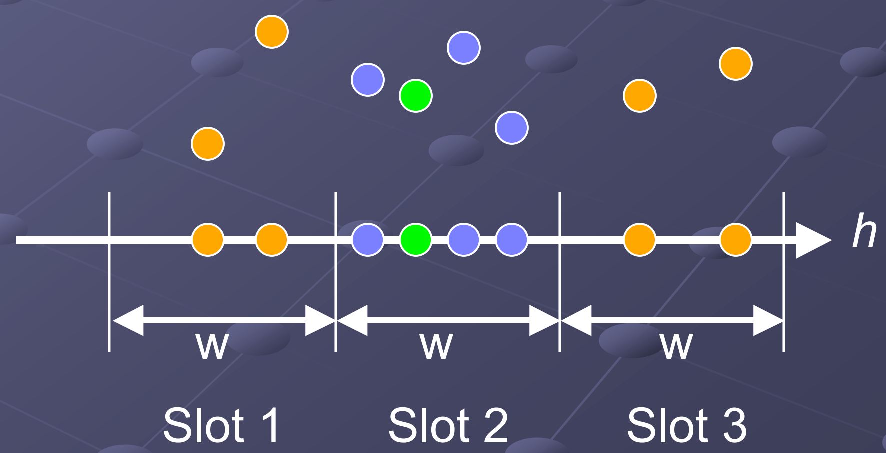
LSH: Locality Sensitive Hashing

- *(r, cr, p_1, p_2)-sensitive* *[IM98]*
  - If $D(q,p) < r$, then $Pr\,[h(q)=h(p)] >= p_1$
  - If $D(q,p) > cr$, then $Pr\,[h(q)=h(p)] <= p_2$
  - *i.e.* closer objects have higher collision probability

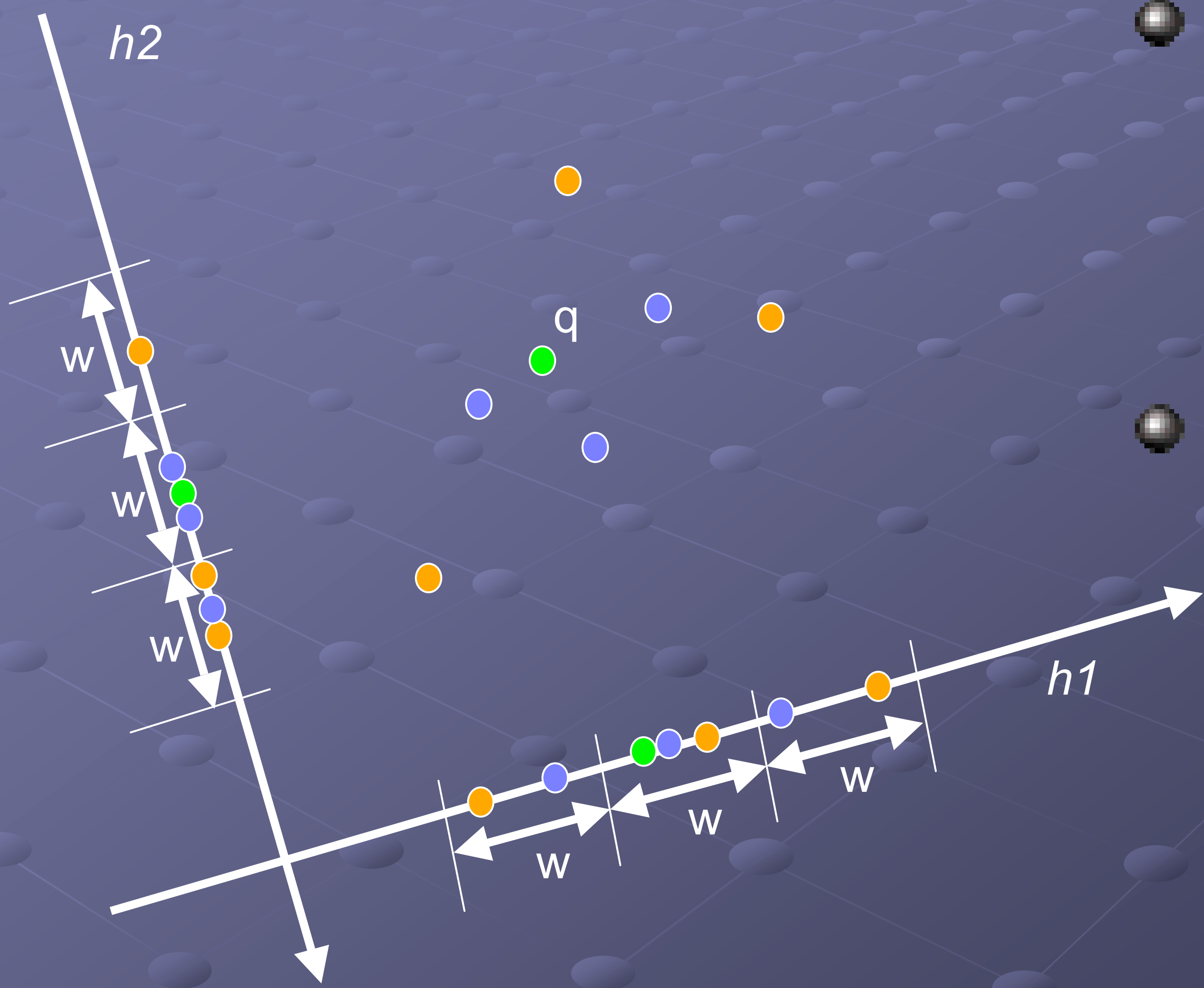


- LSH based on *p*-stable distributions *[DIIM04]*
  - w : slot width

$$h_{a,b}(v) = \left\lfloor \frac{a \cdot v + b}{w} \right\rfloor$$

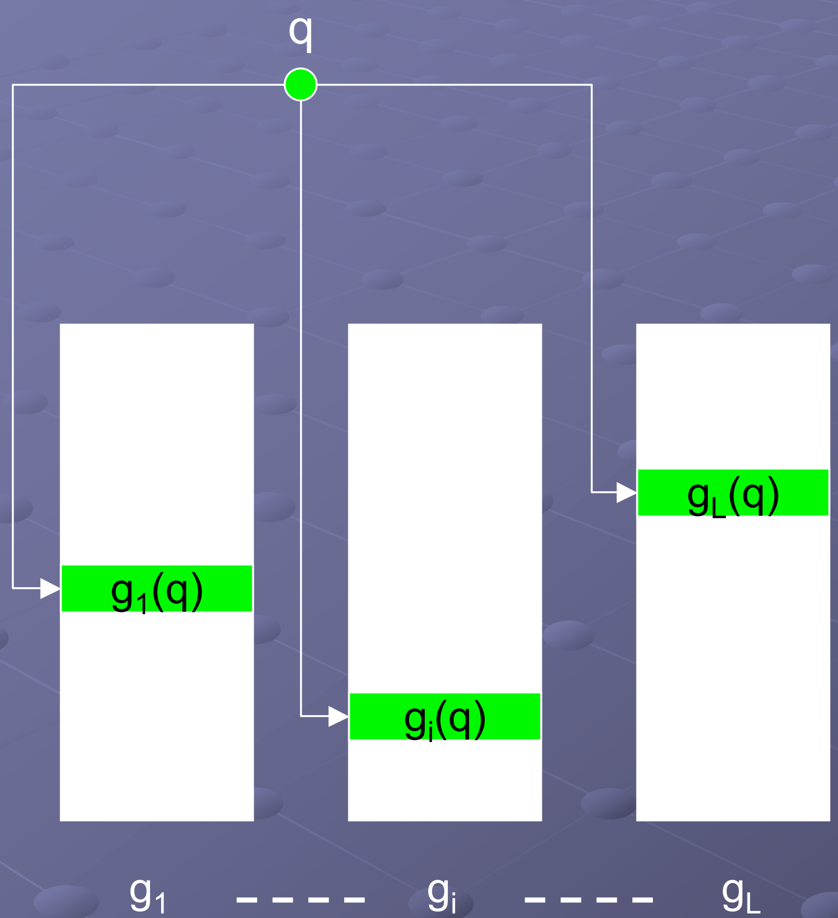


LSH for Similarity Search



- False positive
  - Intersection of multiple hashes
- False negative
  - Union of multiple hashes

Basic LSH Indexing



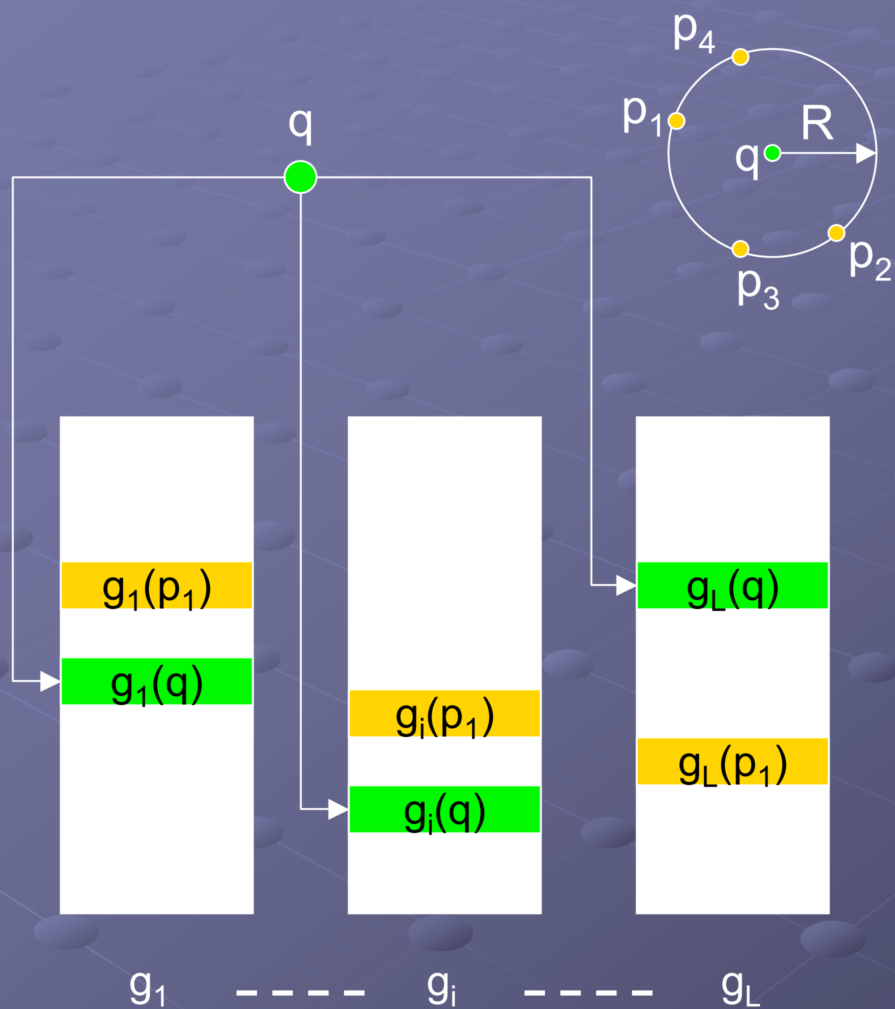
- *[IM98, GIM99, DIIM04]*
- *M* hash functions per table

  $g_i(v) = (\, h_{i,1}(v),\ \ldots,\ h_{i,M}(v)\,)$
- *L* hash tables

  $G = \{\, g_1,\ \ldots,\ g_L \,\}$
- Issues:
  - Large number of tables
  - L > 100 in *[GIM99]*
  - L > 500 in *[Buhler01]*

Impractical for large datasets

Entropy-Based LSH Indexing



- *[Panigrahy, SODA'06]*
- Randomly perturb *q* at distance *R*
- Check hash buckets of perturbed points
- Issues:
  - Difficult to choose *R*
  - Duplicate buckets

Inefficient probing

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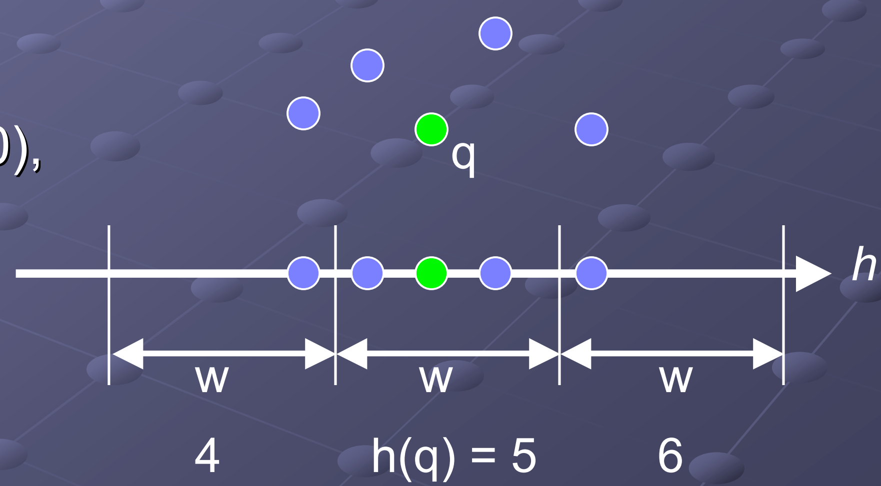
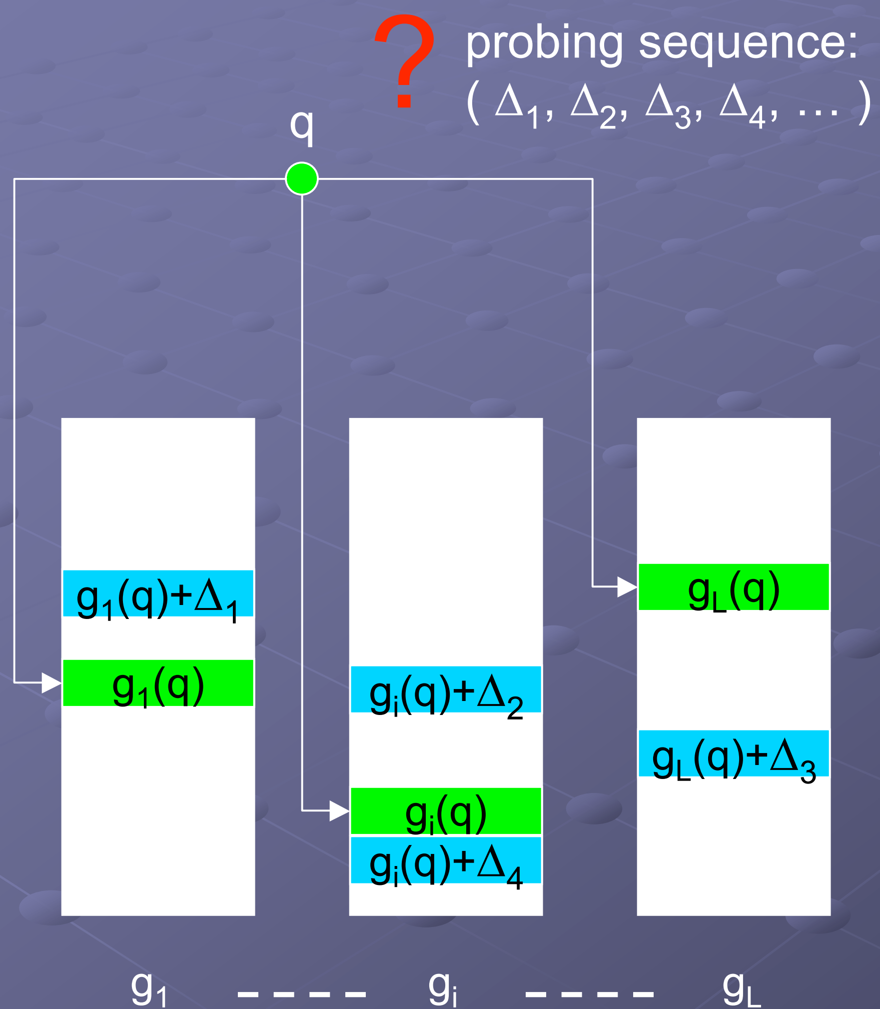
Multi-Probe LSH Indexing

- Probes multiple hash buckets per table
- Perturbs directly on hash values
  - Check left and right slots
  - Perturbation vector Δ
    - $g(q) = (2, 5, 3)$, $\Delta = (-1, 1, 0)$,
    - $g(q) + \Delta = (1, 6, 3)$
- Systematic probing
  - $(\Delta_1, \Delta_2, \Delta_3, \Delta_4, \ldots)$



Multi-Probe LSH Indexing



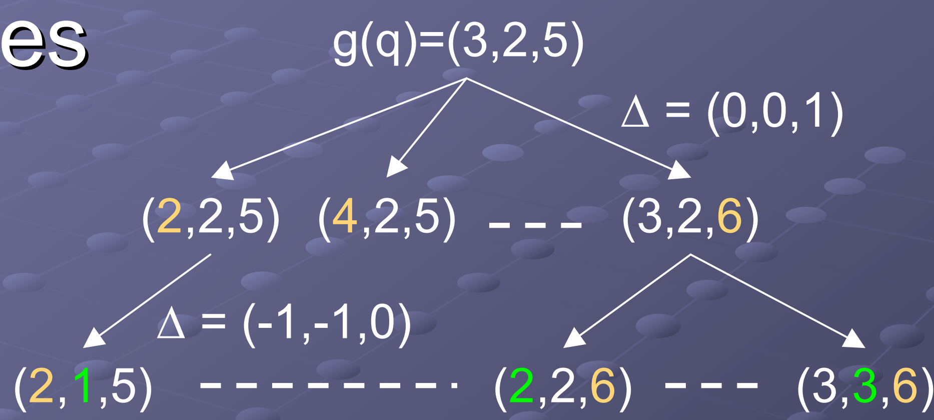
- A carefully derived probing sequence
- Advantages
  - Fast probing sequence generation
  - No duplicate buckets
  - More effective in finding similar objects

Step-Wise Probing

- Given *q*'s hash values

1-step buckets

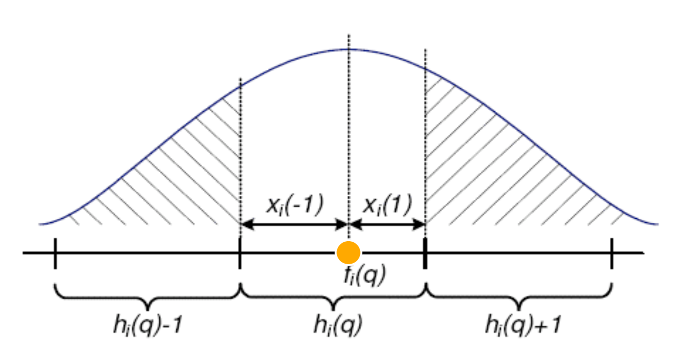
2-step buckets



- Intuitions WRONG!
  - 1-step buckets better than 2-step buckets
  - All 1-step buckets are equally good

Success Probability Estimation

- Hashed position within slot matters!

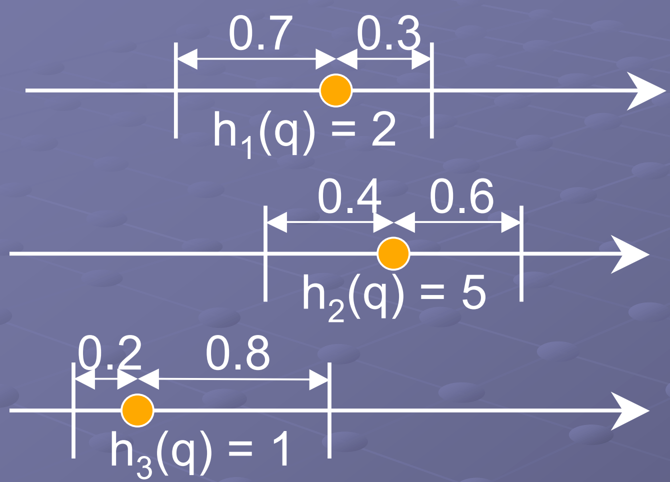


- Estimation based on $x_i(-1)$ and $x_i(1)$

$$Pr[g(p) = g(q) + \Delta] = \prod_{i=1}^{M} Pr[h_i(p) = h_i(q) + \delta_i]$$
$$\approx \prod_{i=1}^{M} e^{-Cx_i(\delta_i)^2} = e^{-C\sum_i x_i((\delta_i)^2)}$$

$$score(\Delta) = \sum_{i=1}^{M} x_i(\delta_i)^2$$

Query-Directed Probing



$g(q) = (h_1(q), h_2(q), h_3(q)) = (2, 5, 1)$

{ 0.2, 0.3, 0.4, 0.6, 0.7, 0.8 }

{ $x_3(-1)$, $x_1(1)$, $x_2(-1)$, $x_2(1)$, $x_1(-1)$, $x_3(1)$ }

{ 0.2 } → { 0.2, 0.3 } → { 0.2, 0.3, 0.4 }

{ 0.2, 0.3 } ↓ { 0.2, 0.4 }

{ 0.2 } ↓ { 0.3 } → { 0.3, 0.4 }

{ 0.3 } ↓ { 0.4 }

{ 0.2 }	$\Delta_1 = (0, 0, -1)$	(2, 5, 0)
{ 0.3 }	$\Delta_2 = (1, 0, 0)$	(3, 5, 1)
{ 0.2, 0.3 }	$\Delta_3 = (1, 0, -1)$	(3, 5, 0)

$$score(\Delta) = \sum_{i=1}^{M} x_i(\delta_i)^2$$

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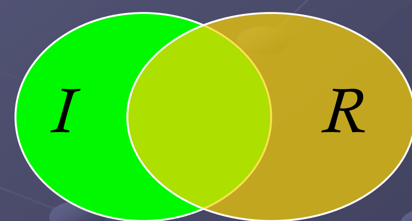
Evaluations

- Multi-probe vs. basic vs. entropy-based
  - Tradeoff among space, speed and quality
  - Space reduction
- Query-directed vs. step-wise probing
  - Tradeoff between search quality and number of probes

Evaluation Methodology

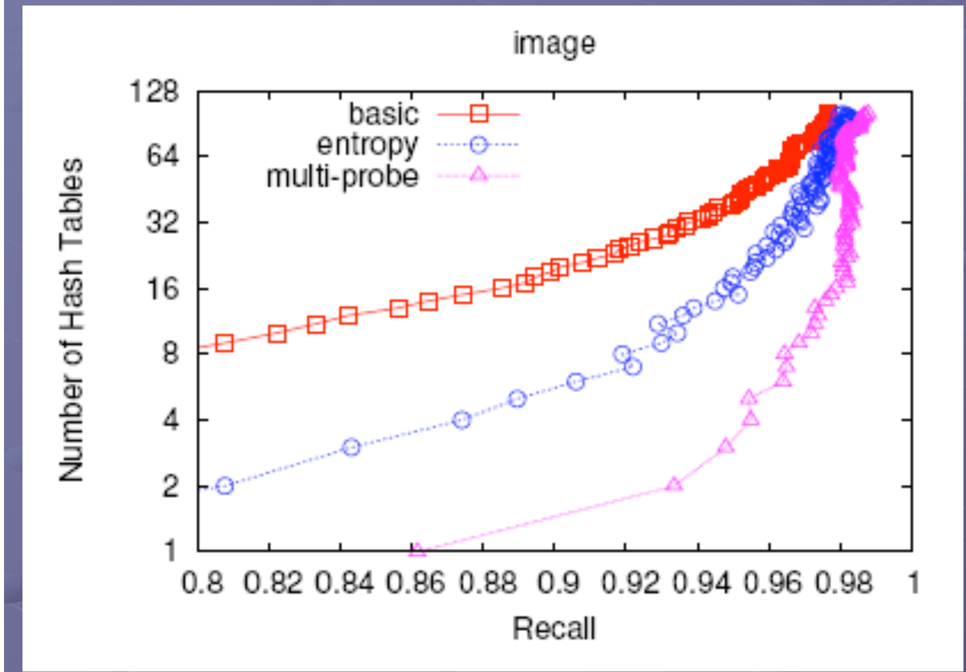
Dataset	#objects	#dimensions
Web images	1.3 million	64
Switchboard audio	2.6 million	192

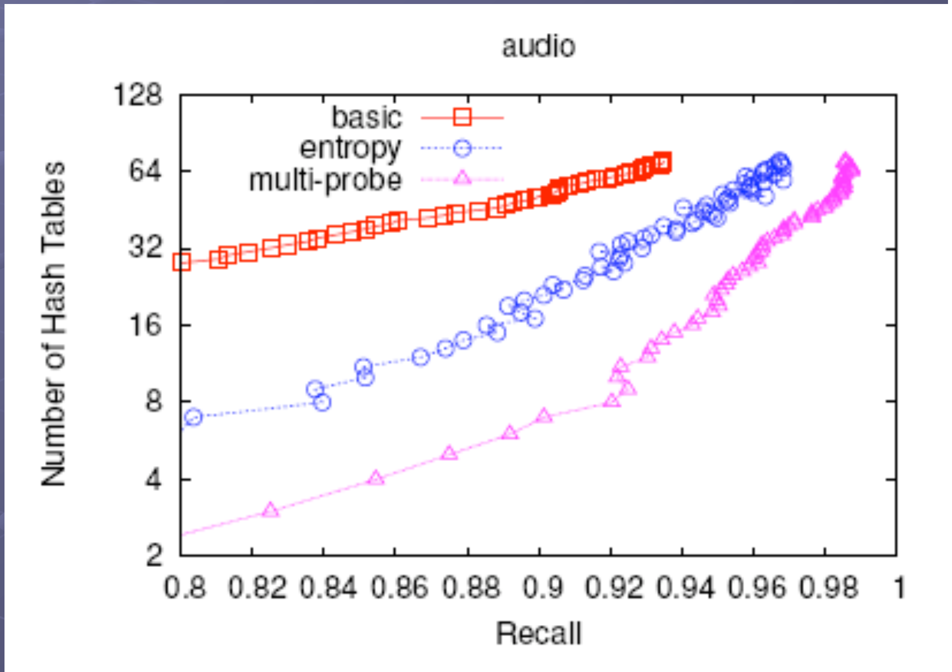
- Benchmarks
  - 100 random queries, top K results
- Evaluation metrics
  - Search quality: recall, error ratio
  - Search speed: query latency
  - Space usage: #hash tables



$$\text{recall} = |I \cap R| \; / \; |I|$$

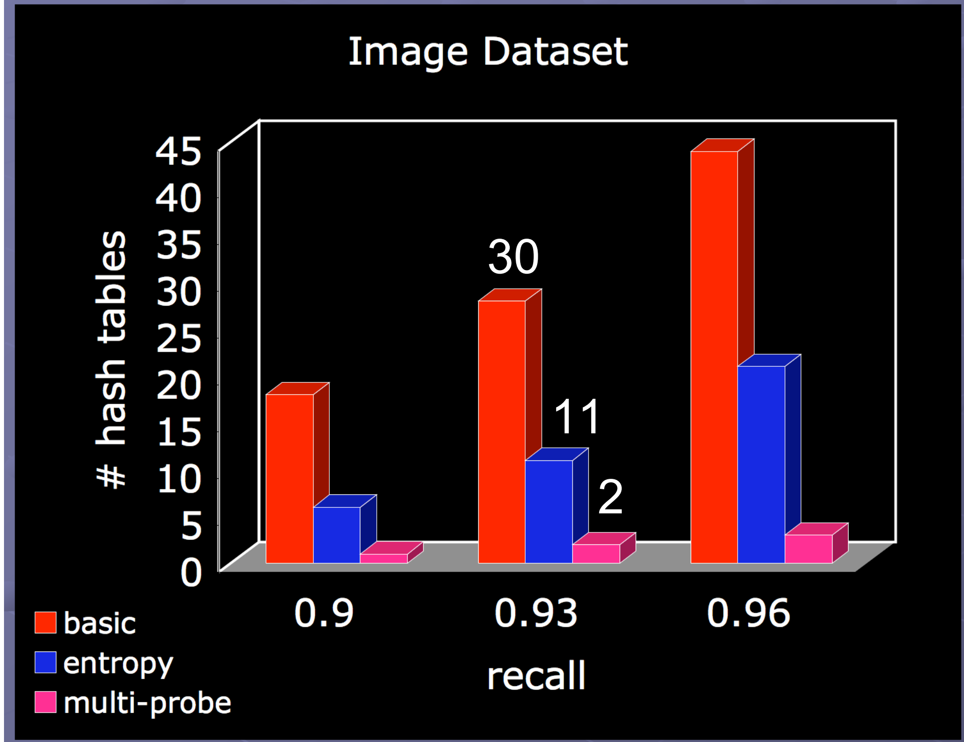
Multi-Probe vs. Basic vs. Entropy

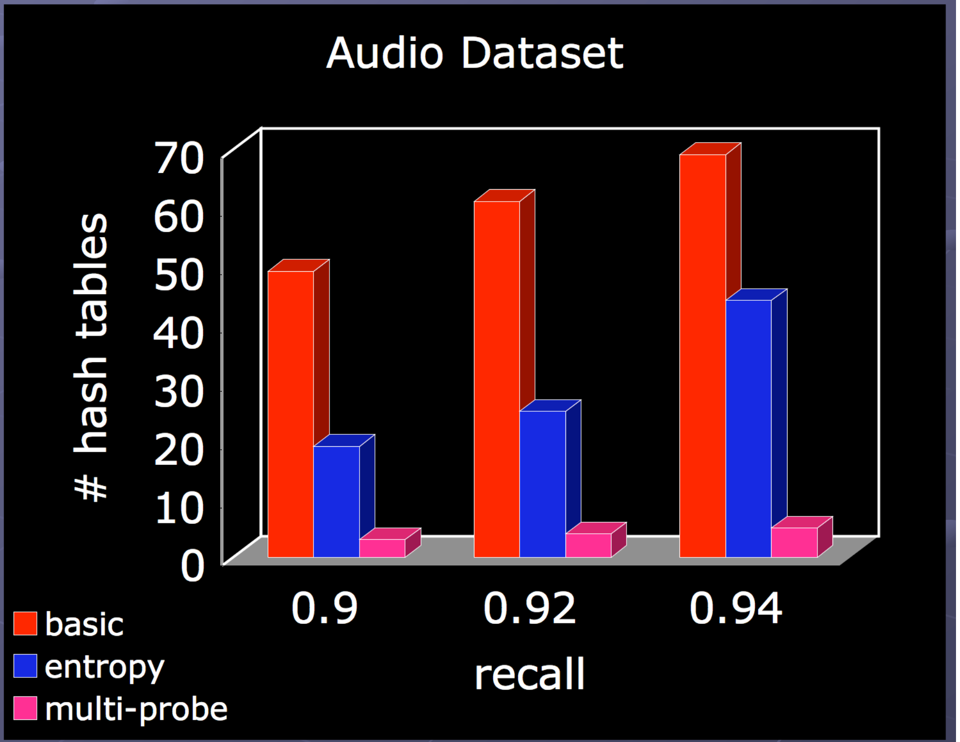




Multi-probe LSH achieves higher recall with fewer hash tables

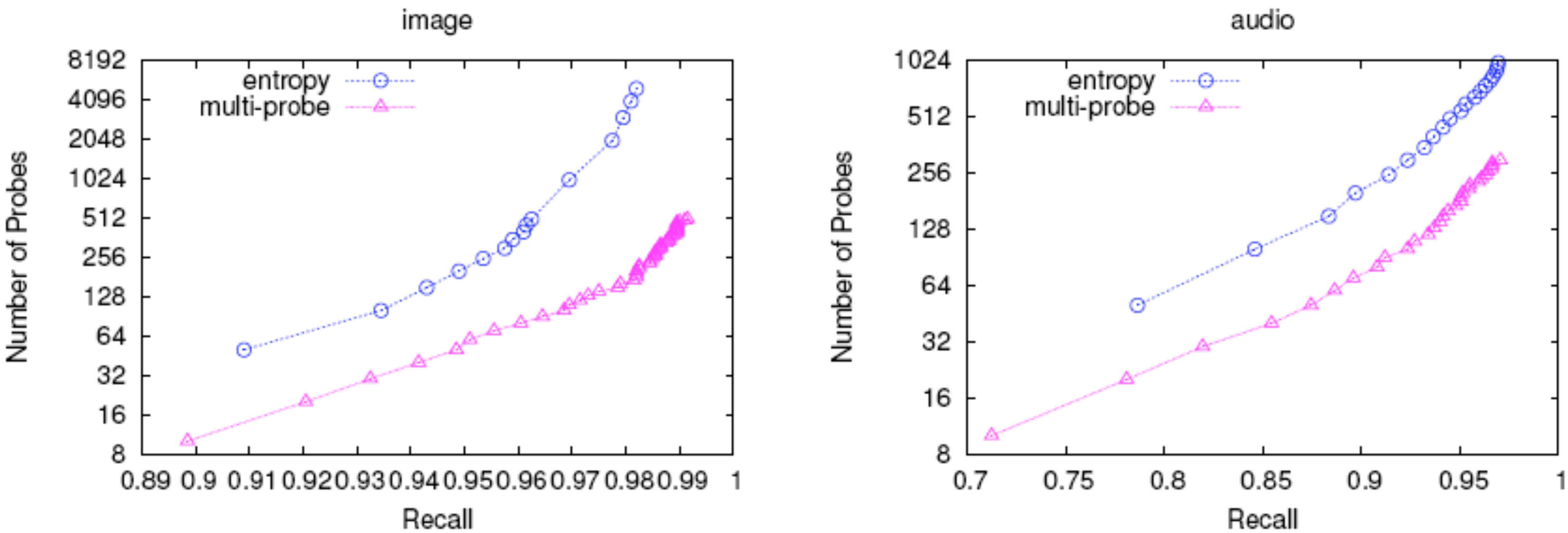
Space Savings of Multi-Probe LSH





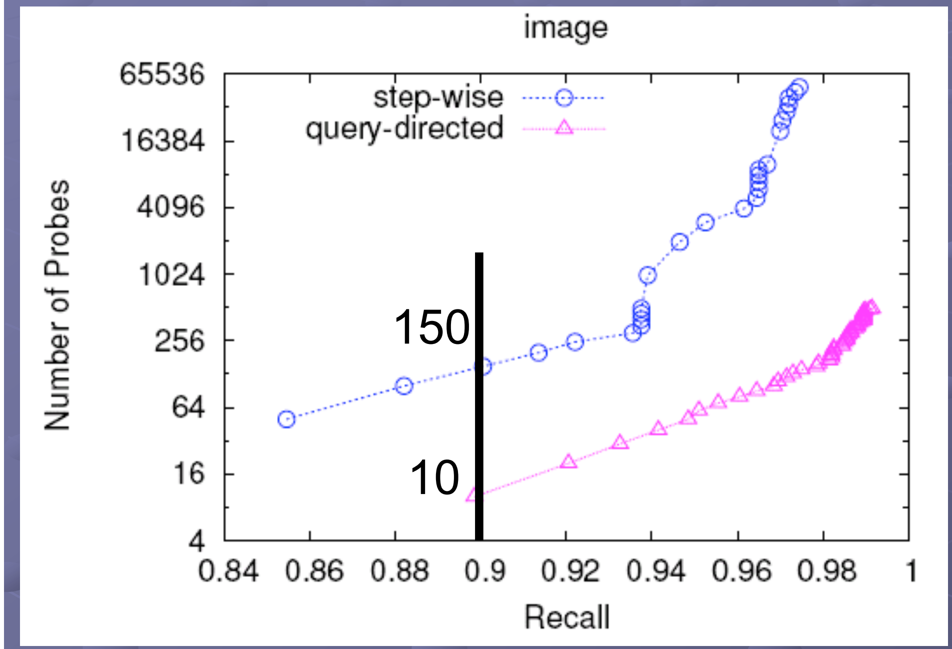
14x - 18x fewer tables than basic LSH
5x - 8x fewer tables than entropy LSH

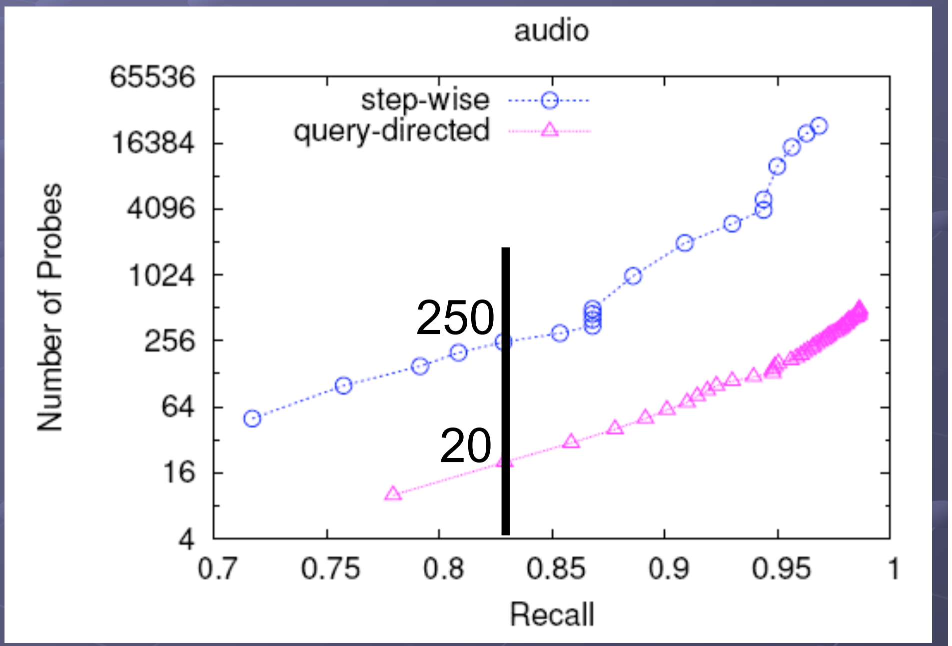
Multi-Probe vs. Entropy-Based



Multi-probe LSH uses much fewer number of probes

Query-Directed vs. Step-Wise Probing





Query-directed probing uses 10x fewer number of probes

Conclusions

- Multi-probe LSH indexing
  - Systematically probes multiple buckets per hash table
  - More space-efficient than basic LSH (14x-18x) and entropy-based LSH (5x-8x)
  - More time-efficient than entropy-based LSH
    - 10x fewer number of probes
  - Query-directed probing is far superior to step-wise probing

Future Work

- Multi-probe LSH on larger datasets
  - 60 million images, out-of-core, distributed
- Self-tuning
  - Analytical model, LSH Forest
- Compare with other indexing methods
- Evaluate on other data types, features
  - Genomic data, video data, scientific sensor data …

Thanks!

- Princeton CASS Project
  - Content-Aware Search Systems
  - http://www.cs.princeton.edu/cass/
- Qin (Christine) Lv at Stony Brook
  - http://www.cs.sunysb.edu/~qlv